

Cosmological Perturbations in the Projectable Version of Hořava-Lifshitz Gravity

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We consider linear perturbations about a homogeneous and isotropic cosmological background in the projectable version of Hořava-Lifshitz gravity. Starting from the action for cosmological perturbations, we identify the canonically normalized fluctuation variables. We find that - in contrast to what happens in the non-projectable version of the theory - the extra scalar cosmological perturbation mode is already dynamical at the level of linear perturbations. For values of the parameter λ in the range $1/3 < \lambda < 1$, the extra mode is ghost-like, for values $1 < \lambda$ and $\lambda < 1/3$ it is tachyonic. This indicates a problem for the projectable version of Hořava-Lifshitz gravity.

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I. OVERVIEW

It has been about one year and a half since Petr Hořava's seminal paper first appeared on the arXiv [1] (see also [2, 3]). Hořava proposes a model for quantum gravity which is based on an anisotropic scaling between space and time, and on constructing the most general power-counting renormalizable Lagrangian with respect to this new scaling. This construction involves an explicit violation of both general covariance and Lorentz invariance. The construction is modeled after a class of condensed-matter models exhibiting anisotropic scaling whose prototype is the theory of a Lifshitz scalar [4]. Hence, the theory is now called "Hořava-Lifshitz gravity".

The basic fields in Hořava-Lifshitz (HL) gravity are the same as in Einstein gravity. However, there are less symmetries. Hence, it is expected that an extra dynamical degree of freedom will emerge. Such a degree of freedom would likely cause serious problems for the theory. In particular, it could be ghost-like or tachyonic, both of which would be quite problematic. Indeed, when expanding about Minkowski space-time, an extra degree of freedom in the scalar gravitational sector of the theory appears [1]. Rather surprisingly, it was found [5, 6] that in the "non-projectable" version of the theory, the extra scalar gravitational degree of freedom is non-dynamical when expanding about a homogeneous and isotropic cosmological background¹. There are reasons (which will be mentioned in the following section), however, to prefer the "projectable" version of Hořava-Lifshitz gravity. The structure of the constraints is very different in the two versions of the theory, and hence it is interesting to study linear cosmological perturbation theory in the case of the "projectable" version of HL gravity. This is what we do in this paper, and we find that the extra scalar mode of the linear cosmological perturbations becomes dynamical. The mode is either ghost-like or tachyonic depending on the value of the parameter λ which enters into the kinetic term of the Hořava-Lifshitz Lagrangian.

The outline of this paper is as follows: in the following section we give an introduction to the problem and survey previous works. In Section 3 we review Hořava-Lifshitz gravity and the cosmological background solutions which the theory admits. Then follows the main section of this paper in which we study linear cosmological perturbations in the projectable version of the theory, identify the dynamical variables and study under which conditions the extra degree of freedom is ghost-like or tachyonic.

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¹ There are, however, reasons to expect that the extra mode will appear beyond linear order in perturbation theory [7].

II. INTRODUCTION AND REVIEW

Hořava-Lifshitz gravity is based on the assumption that space and time have different scaling behaviors with respect to some renormalization group flow. The flow is defined by a critical exponent z :

$$\begin{aligned} x^i &\rightarrow b x^i \\ t &\rightarrow b^z t \end{aligned} \tag{2.1}$$

In order to obtain gravity in four dimensional space-time we set $z = 3$ so that the theory can be power-counting renormalizable, for reasons that we will point out later in this paper. Lorentz invariance is thus explicitly broken if $z \neq 1$, but it is desired that z flow to 1 in the infra-red, thereby restoring Lorentz symmetry at low energies. However, it has not yet been shown explicitly that this will occur (see [8–10] for a study of the renormalization group flow in scalar field theories of Lifshitz type).

Different versions of HL gravity can be formulated depending on assumptions other than power-counting renormalizability and symmetry under both spatial diffeomorphisms and space-independent time reparametrizations which are added. In particular, one can impose the requirement of “detailed balance” on the potential terms in the Lagrangian. Detailed balance was invoked by Hořava in his seminal work as a simplifying assumption which reduces the number of terms and thus simplifies the algebra. However, there are no particular physical reasons to make this assumption, and it was also realized that a Lagrangian with detailed balance leads to phenomenological problems (see e.g. [11–14]).

A more important criterion is “projectability”. If this condition is imposed, then the lapse function is independent of space, if not, then it is allowed to depend on space. Whether or not the projectability condition should be maintained is a question which still awaits further investigation. We mention that the projectability condition is supported in [15] because it prevents a non-relativistic theory of gravity from developing inconsistencies, whereas, on the other side, it is opposed in [14] because it gives rise to a non-local Hamiltonian constraint, and thus appears to make it harder to recover General Relativity (in which the constraint is local) in the infrared limit². Another reason in favor of imposing the projectability condition is that the algebra of constraints appears consistent only if this condition is imposed [17]. The reader is referred to [18] for a more detailed discussion of these points. More recently, a “healthy” extension of HL gravity was proposed [19], an extension which may resolve some of the key challenges the original formulations face.

HL gravity has attracted the attention of cosmologists for several reasons. It may lead to new solutions of some old cosmological problems (see e.g. [11, 20–22]) by providing alternatives to the inflationary paradigm of the very early universe. In particular, in a spatially curved background a bouncing cosmology is obtained naturally [23], (see also [24, 25]) and, with the addition of scalar field matter, HL gravity can yield a scale-invariant spectrum of curvature perturbations in the ultra-violet limit even for non-inflationary expansion of the background space-time [21].

HL gravity starts out with the same number of dynamical degrees of freedom as General Relativity, but has less symmetries. Specifically, one loses one gauge symmetry, namely space-dependent time rescalings. Hence, we should expect an extra degree of freedom to arise [1]. The emergence of this extra degree of freedom has been studied in a number of works when expanding about flat space-time, i.e. in the absence of matter (see e.g. [13, 14, 18, 26–33]), both in the projectable and in the non-projectable versions of the theory. The extra mode is not only a problem phenomenologically (no such mode has been observed), but there are also conceptual problems: the mode is either tachyonic (for values of λ in the ranges $\lambda < 1/3$ and $1 < \lambda$) or ghost-like (for $1/3 < \lambda < 1$)³.

There have been fewer analyses of the extra mode in the presence of matter. Interestingly, it was found in [5] that in the non-projectable version of the theory, the extra scalar gravitational mode is not dynamical when expanding about a homogeneous and isotropic metric in the presence of matter⁴. This conclusion holds both for flat and for curved spatial sections [6]. In this paper we will show that in the projectable version of HL gravity the extra mode is present, and that it has the same tachyonic and ghost-like properties as a function of λ as it does in the absence of matter when expanding about Minkowski space-time.

The conclusions we reach are the same as those reached recently in [37]. In that paper, the constraint algebra of linear cosmological perturbation theory about an expanding cosmological background was analysed, and it was shown why in the non-projectable version of HL gravity there is only one dynamical degree of freedom for adiabatic scalar fluctuations, whereas two remain in the projectable version. Our analysis reaches the same conclusions, but

² Note, however, that a nonlocal constraint can lead to interesting consequences for cosmology [16].

³ Note that phenomenological aspects of the spectrum of gravitational waves [34] and scalar metric fluctuations [35, 36] have also been studied.

⁴ However, the later work of [7] indicates that the dangerous extra mode will appear at higher order in perturbation theory, and also at linear order if the background is not maximally symmetric.

uses a slightly different method. It follows that of [5], in that we work out the quadratic action for linear scalar cosmological perturbations, identify the dynamical degrees of freedom which have canonical normalization, and study the coefficient matrices of both the kinetic part of the Lagrangian and of the mass terms in order to identify possible ghost-like or tachyonic states.

There are other concerns about fluctuations in HL gravity. One is the issue of strong coupling - which would spoil the recovery of General Relativity in the infra-red limit - (see [14]). In the infrared limit (which needs to correspond to $\lambda \rightarrow 1$ if the theory is to be phenomenologically viable) the couplings of the extra degree of freedom diverge in the limit $\lambda \rightarrow 1$ and thus indicate that cosmological perturbation theory breaks down. This issue is controversial even in light of the “healthy extension” proposed in [19] to cure this and other problems of HL gravity (this is further discussed in [38, 39]). Once again, the problem is absent in linear cosmological perturbation theory in the case of the non-projectable version of HL gravity [5]. We will show that the problem reappears in the projectable version.

In this paper we will consider cosmological perturbations about a spatially flat Friedmann-Robertson-Walker (FRW) background in the projectable version of HL gravity, without detailed balance, and in the presence of scalar field matter. We work out the second order action for scalar metric perturbations and study the number of physical degrees of freedom and their ghost/tachyonic instabilities. The results will be compared to the ones in [5], where a similar study was done in the case of the non-projectable version of HL gravity (also in the absence of detailed balance), and where it was concluded that the extra gravitational degree of freedom which is dangerous when expanding about flat space-time is non-dynamical, that the limit $\lambda \rightarrow 1$ - in which General Relativity is supposed to be recovered - is smooth, and that no strong-coupling problem arises. We find that all of these nice features disappear in the projectable version⁵.

III. SETUP

A. Hořava-Lifshitz Gravity

Hořava-Lifshitz gravity is based on the same metric variables as General Relativity. The symmetry group of the theory is assumed to be different. The theory is not invariant under the full space-time diffeomorphism group, but only under the restricted group of spatial diffeomorphisms and space-independent time reparametrizations. Instead, the anisotropic scaling symmetry of (2.1) is postulated.

The anisotropy between space and time is readily taken into account if we adopt the Arnowitt-Deser-Misner (ADM) decomposition of the metric [41]:

$$ds^2 = -N^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (3.1)$$

where t is physical time, the x^i coordinates ($i = 1, \dots, 3$) are comoving spatial coordinates, and g_{ij} is the metric on the constant time hypersurfaces. The gravitational dynamical degrees of freedom are the lapse function N , the shift vector N^i and the spatial metric g_{ij} . In principle N , N^i and g_{ij} can be functions of both space and time, unless one imposes extra restrictive conditions. In particular, the “projectability condition” forces N be a function of time only, i.e. $N \equiv N(t)$.

The construction of the most general action of Hořava-Lifshitz gravity relies on the requirement that the theory be power-counting renormalizable with respect to the scaling symmetry. As reviewed e.g. in [42], to obtain the HL version of gravity in d spatial dimensions, we need to set $z = d$ (see also [43] for a more general analysis). Given the choice of z , one builds up the action by adding all terms which are renormalizable or super-renormalizable, and which are consistent with the residual symmetries which are imposed. The whole procedure is very clearly described in [18] and [44] and will be only briefly summarized in what follows.

Denoting the scaling dimensions with square brackets and a “ s ” subscript, we have

$$[t]_s = -z, \quad [x^k]_s = -1 \quad (3.2)$$

as can be seen from Eq. (2.1). We then require the action to be dimensionless. Note that

$$[S]_s = \left[\int dt d^d x \mathcal{L} \right]_s = 0 \iff [\mathcal{L}]_s = z + d. \quad (3.3)$$

⁵ In work in progress, we are studying the same questions for the “safe” version of HL gravity of [19], a version for which some initial work on cosmological fluctuations has recently been reported in [38, 40].

In $3 + 1$ dimensions the scaling dimension of the Lagrangian must be equal to six, meaning that the Lagrangian is expected to include terms with more spatial derivatives terms than those which appear in General Relativity. The scaling dimensions of the metric coefficients are

$$[g_{ij}]_s = 0, \quad [N^i]_s = z - 1, \quad [N]_s = 0. \quad (3.4)$$

Note that the mass dimensions of both space and time coordinates are still equal to -1 and hence mass and scaling dimensions do not coincide.

As in [33, 35], we consider the following action:

$$S = \chi^2 \int dt d^3x N \sqrt{g} (\mathcal{L}_K - \mathcal{L}_V + \chi^{-2} \mathcal{L}_M) \quad (3.5)$$

where $g \equiv \det(g_{ij})$, $\chi^2 \equiv 1/(16\pi G)$ and the kinetic, potential and matter Lagrangians are, respectively, given by

$$\mathcal{L}_K = K_{ij} K^{ij} - \lambda K^2 \quad (3.6a)$$

$$\begin{aligned} \mathcal{L}_V &= 2\Lambda - R + \frac{1}{\chi^2} (g_2 R^2 + g_3 R_{ij} R^{ij}) + \frac{1}{\chi^4} \left(g_4 R^3 + g_5 R R_{ij} R^{ij} + g_6 R_j^i R_k^j R_i^k \right) + \\ &+ \frac{1}{\chi^4} [g_7 R \nabla^2 R + g_8 (\nabla_i R_{jk}) (\nabla^i R^{jk})] \end{aligned} \quad (3.6b)$$

$$\mathcal{L}_M = \frac{1}{2N^2} (\dot{\varphi} - N^i \nabla_i \varphi)^2 - V(g_{ij}, \mathcal{P}_n, \varphi), \quad (3.6c)$$

and the extrinsic curvature K_{ij} is defined as

$$K_{ij} = \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \quad (3.7)$$

and, as customary, dots over variables represent time derivatives. Indices are raised and lowered using the spatial metric g_{ij} and all the terms in the action depending on the Ricci tensor and its contractions are built from the same spatial metric. The scalar field Lagrangian differs from that in theories with full general covariance in that it can also contain terms with higher spatial derivatives as long as they do not destroy the power-counting renormalizability of the theory with respect to the scaling symmetry. Thus, the non-kinetic part of the scalar field Lagrangian can be written as follows [33, 35, 44]:

$$V = V_0(\varphi) + V_1(\varphi)\mathcal{P}_0 + V_2(\varphi)\mathcal{P}_1^2 + V_3(\varphi)\mathcal{P}_1^3 + V_4(\varphi)\mathcal{P}_2 + V_5(\varphi)\mathcal{P}_0\mathcal{P}_2 + V_6(\varphi)\mathcal{P}_1\mathcal{P}_2, \quad (3.8)$$

where

$$\mathcal{P}_0 \equiv (\nabla \varphi)^2, \quad \mathcal{P}_i \equiv \Delta^i \varphi, \quad \Delta \equiv g^{ij} \nabla_i \nabla_j. \quad (3.9)$$

Some remarks are in order:

1. If one introduces the ADM metric (3.1) into the Einstein-Hilbert action,

$$S_{EH} = \chi^2 \int d^4x \sqrt{-g} {}^{(4)}R \quad (3.10)$$

where ${}^{(4)}R$ is the four-dimensional Ricci scalar, the resulting kinetic Lagrangian will have the same form as (3.6a) with $\lambda = 1$, whereas the potential Lagrangian would simply include the term $\propto R$ (see for instance [45]).

2. Note that there are several coupling constants entering the potential contribution \mathcal{L}_V to the Lagrangian, namely χ and the g 's. The coupling constants g_I , $I = (2, \dots, 8)$, have zero mass dimension, but non-zero scaling dimensions, while χ is dimensionless with respect to the scaling symmetry but has energy dimension equal to one. We further observe that the couplings to Λ and R are set to unity. These two terms are super-renormalizable with respect to the scaling symmetry.

Varying the action with respect to $N(t)$ one can derive the following Hamiltonian constraint:

$$\int d^3x \sqrt{g} \left(\mathcal{L}_K + \mathcal{L}_V - \frac{1}{2\chi^2} J^t \right) = 0, \quad (3.11)$$

where

$$J^t \equiv 2 \left(N \frac{\delta \mathcal{L}_M}{\delta N} + \mathcal{L}_M \right). \quad (3.12)$$

The Hamiltonian constraint is non-local in the projectable version of HL gravity (it is expressible as a volume integral instead of being local).

The variation of the action with respect to $N^i(t, x^k)$ results in the following super-momentum constraint:

$$\nabla_i \pi^{ij} = \frac{1}{2\chi^2} J^j, \quad (3.13)$$

where the super-momentum π^{ij} and the matter current are given, respectively, by:

$$\pi^{ij} \equiv \frac{\delta \mathcal{L}_K}{\delta \dot{g}_{ij}} = -K^{ij} + \lambda K g_{ij} \quad (3.14)$$

and

$$J_i \equiv -N \frac{\delta \mathcal{L}_M}{\delta N^i} = \frac{1}{N} (\dot{\varphi} - N^k \nabla_k \varphi) \nabla_i \varphi. \quad (3.15)$$

We will make explicit both constraints in the following section.

B. Cosmological Background

The metric in the ADM form as in Eq. (3.1) can be reduced to a spatially flat FRW metric if one sets the values of N , N^i and g_{ij} as follows:

$$\begin{aligned} N(t) &= 1 + \delta N(t), \\ N^i(t, x^k) &= 0 + \delta N^i(t, x^k), \\ g_{ij}(t, x^k) &= a^2(t) \delta_{ij} + \delta g_{ij}(t, x^k). \end{aligned} \quad (3.16)$$

The perturbations δN , δN^i , δg_{ij} will be specified later on.

On evaluating the Hamiltonian constraint Eq. (3.11) to zeroth order one obtains

$$(3\lambda - 1) H^2 = \frac{1}{3} \left(\frac{\rho_M}{\chi^2} + 2\Lambda \right) \quad (3.17)$$

which generalizes the first Friedmann equation. Here, ρ_M is the energy density associated to the matter sector. Note that in the absence of Λ , H^2 is strictly positive only for $\lambda > 1/3$. Otherwise, if $\lambda < 1/3$, it is positive only in presence of a sufficiently negative cosmological constant ($\Lambda < -\rho_M/M_{\text{Pl}}$). However, the range $\lambda < 1/3$ is not interesting phenomenologically because it is disconnected by the singular point $\lambda = 1/3$ from the value $\lambda = 1$ for which one wishes to recover General Relativity.

The dynamical equation for the scale factor $a(t)$, the generalization of the second Friedmann equation, can be obtained by varying the action with respect to g_{ij} and evaluating the result in the homogeneous limit. The result is:

$$(3\lambda - 1) \frac{\ddot{a}}{a} = -\frac{1}{6\chi^2} (\rho_M + 3p_M) + \frac{2}{3} \Lambda, \quad (3.18)$$

where p_M is the (background) pressure associated with matter. For scalar field matter we have

$$\begin{aligned} \rho_M &= \frac{\dot{\varphi}_0^2}{2} + V_0(\varphi_0), \\ p_M &= \frac{\dot{\varphi}_0^2}{2} - V_0(\varphi_0), \end{aligned} \quad (3.19)$$

and the background equation of motion becomes:

$$\ddot{\varphi}_0 + 3H\dot{\varphi}_0 = -\frac{dV_0(\varphi_0)}{d\varphi_0} \quad (3.20)$$

exactly as in General Relativity.

IV. COSMOLOGICAL PERTURBATIONS

A. Introduction

In this section we are interested in scalar metric fluctuations, the fluctuation modes which couple to energy density and pressure. Vector and tensor perturbations are studied in [37]. The basic fluctuation variables are the same as in the case of General Relativity (see e.g. [46] for an in-depth review of the theory of cosmological fluctuations and [47] for an introductory overview):

$$\delta N(t) = \phi(t) \quad (4.1a)$$

$$\delta N_i(t, x^k) = \partial_i B(t, x^k) \quad (4.1b)$$

$$\delta g_{ij}(t, x^k) = a^2(t) [-2\psi(t, x^k) \delta_{ij} + 2E(t, x^k)_{|ij}] \quad (4.1c)$$

where the subscript $|_i$ denotes the covariant derivative. Correspondingly, also matter fluctuations must be taken into account:

$$\varphi(t, x^k) = \varphi_0(t) + \delta\varphi(t, x^k). \quad (4.2)$$

The first major difference (particular to the projectable version of HL gravity) compared to Einstein gravity is that the variable ϕ depends only on time. The second major difference concerns the symmetry group. In the case of HL gravity it is reduced compared to the case of Einstein gravity. One loses the space-dependent time reparametrizations. In the case of General Relativity one can use the two scalar gauge degrees of freedom to set $E = B = 0$. In HL gravity there is only one space-dependent gauge mode. One can use this mode to realize the gauge choice $E = 0$. In the projectable version of HL gravity one can in addition make use of space-independent time reparametrizations to set $\phi = 0$ (in the non-projectable version of the theory one cannot make this choice since ϕ then can depend non-trivially on space). In conclusion, in our work can use the gauge freedom to set

$$\phi = 0, \quad E = 0, \quad (4.3)$$

which corresponds to the choice of the quasi-longitudinal gauge (see also [33, 35]).

Expanding the Hamiltonian constraint (3.11) to first order one finds

$$\int d^3x a^3 \left[2\Delta\psi - (3\lambda - 1)H(\Delta B + 3\dot{\psi}) - \frac{\delta\rho_M}{2\chi^2} \right] = 0, \quad (4.4)$$

which is actually trivially satisfied when dealing with linear cosmological perturbations since the spatial average of linear fluctuations must vanish (any non-vanishing term would be a contribution to the background solution). Note at this stage a key difference compared to the non-projectable version of HL gravity: in the latter case the linear Hamiltonian constraint is local and hence provides non-trivial constraints.

In contrast, the super-momentum constraint (3.13) is trivial at the homogeneous level but not at first order where it becomes

$$\partial_j \left[(\lambda - 1)\Delta B + (3\lambda - 1)\dot{\psi} - \frac{1}{2\chi^2}q_M \right] = 0, \quad (4.5)$$

with

$$q_M = \dot{\varphi}_0 \delta\varphi. \quad (4.6)$$

In linear perturbation theory we can work in Fourier space where the spatial derivative ∂_i can be replaced by $(-i k_j)$. Hence, the quantities inside the square brackets of (4.5) must sum to zero.

Note that we started with five scalar degrees of freedom as in Eqs. (4.1), (4.2), and then we have decreased their number by two by making use of the gauge freedom. The number of degrees of freedom can be further reduced by one using the first order super-momentum constraint (4.5) to remove B and thus only two degrees of freedom - ψ and $\delta\varphi$ - are left.

B. Second-order action

In the following we will insert the ansatz for cosmological fluctuations of the previous subsection into the action for HL gravity and determine the second order action, the terms quadratic in the fluctuation variables. This will allow us to find the canonically normalized fluctuation variables and determine if they are ghost-like or tachyonic.

The second order action receives three contributions, namely:

$$\delta_2 S^{(s)} = \chi^2 \int dt d^3x \left[\delta_0(\sqrt{g}) \delta_2 \mathcal{L}^{(s)} + \delta_1(\sqrt{g}) \delta_1 \mathcal{L}^{(s)} + \delta_2(\sqrt{g}) \delta_0 \mathcal{L}^{(s)} \right] \equiv \chi^2 \int dt d^3x a^3 \mathcal{L}_2^{(s)}, \quad (4.7)$$

where we have implicitly introduced the following notation to denote the orders in the perturbative expansion:

$$f \equiv \sum_{i=0}^{\infty} \delta_i f. \quad (4.8)$$

Concerning the expansion of \sqrt{g} one readily has

$$\delta_0(\sqrt{g}) = a^3, \quad \delta_1(\sqrt{g}) = -3a^3\psi, \quad \delta_2(\sqrt{g}) = \frac{3}{2}a^3\psi^2. \quad (4.9)$$

After making use of the gauge choice to eliminate ϕ and E , and the constraint equation to express B in terms of the two remaining scalar degrees of freedom, the second order scalar action acquires the following form in terms of ψ and $\delta\varphi$:

$$\begin{aligned} \mathcal{L}_2^{(s)}[\psi, \delta\varphi] = & \frac{4(3\lambda-1)}{(\lambda-1)} \frac{\dot{\psi}^2}{2} + \frac{\dot{\delta\varphi}^2}{2\chi^2} + f_\psi \psi \dot{\psi} + f_{\varphi\psi} \psi \dot{\delta\varphi} + \tilde{f}_{\varphi\psi} \dot{\psi} \delta\varphi + \\ & - m_\psi^2 \psi^2 - m_\varphi^2 \delta\varphi^2 - m_{\varphi\psi}^2 \psi \delta\varphi \\ & + \omega_\varphi \delta\varphi \Delta \delta\varphi + \omega_\psi \psi \Delta \psi \\ & + d_\psi (\Delta \psi)^2 + d_\varphi (\Delta \delta\varphi)^2 + \tilde{d}_\psi \Delta \psi \Delta^2 \psi + \tilde{d}_\varphi \Delta \delta\varphi \Delta^2 \delta\varphi \end{aligned} \quad (4.10)$$

where the various coefficients are listed in Appendix A. In order to obtain this result we did some integrations by parts in intermediate steps and used the background dynamical equations for $a(t)$ and $\varphi_0(t)$.

We observe that the coefficient multiplying $\dot{\psi}^2$ has a “wrong” negative sign for $1/3 < \lambda < 1$, which will give rise to ghost instabilities as reported in almost all the literature about Hořava-Lifshitz gravity (see for instance [18, 22, 28, 32, 33]).

In order to compare our result with the analyses in previous works done in the absence of matter, we can set the matter terms to zero in our result and consider the remaining pieces in the second order action for the extra gravitational scalar degree of freedom which reads as follows:

$$\begin{aligned} \delta_2 S^{(s)}[\psi] = & \int dt d^3x a^3 \left\{ \frac{4(3\lambda-1)}{(\lambda-1)} \frac{\dot{\psi}^2}{2} + 6H(1-3\lambda)\psi \dot{\psi} - 15(1-3\lambda)H^2\psi^2 + \right. \\ & \left. - 2\psi \Delta \psi - (16g_2 + 6g_3) \frac{(\Delta \psi)^2}{\chi^2} + (6g_8 - 16g_7) \frac{\Delta \psi \Delta^2 \psi}{\chi^4} \right\}. \end{aligned} \quad (4.11)$$

This result is very similar to Eq. (33) of Ref. [22] and to Eq. (39) of Ref. [13] once the spatial derivatives are set to zero, except for a discrepancy in the coefficient multiplying $(1-3\lambda)H^2\psi^2$ which is -15 in our result instead of 27 appearing in the cited references.

In order to draw definite conclusions about the ghost nature of the fluctuation modes, we must identify the canonically normalized variables. For values of λ which lie in the regions $\lambda < 1/3$ or $\lambda > 1$ we rescale the fields as

$$\tilde{\psi} \equiv \sqrt{\frac{4(3\lambda-1)}{\lambda-1}} \psi, \quad \tilde{\delta\varphi} \equiv \frac{\delta\varphi}{\chi} \quad (4.12)$$

and obtain the following second order Lagrangian in terms of canonically normalized variables:

$$\begin{aligned} \mathcal{L}_2^{(s)}[\psi, \delta\varphi] = & \frac{1}{2} \tilde{\psi}^2 + \frac{1}{2} \tilde{\delta\varphi}^2 + f_{\tilde{\psi}} \tilde{\psi} \dot{\tilde{\psi}} + f_{\tilde{\varphi}\tilde{\psi}} \tilde{\psi} \dot{\tilde{\delta\varphi}} + \tilde{f}_{\tilde{\varphi}\tilde{\psi}} \tilde{\psi} \dot{\tilde{\psi}} \dot{\tilde{\delta\varphi}} + \\ & - m_{\tilde{\psi}}^2 \tilde{\psi}^2 - m_{\tilde{\varphi}}^2 \tilde{\delta\varphi}^2 - m_{\tilde{\varphi}\tilde{\psi}}^2 \tilde{\psi} \tilde{\delta\varphi} \\ & + \omega_{\tilde{\varphi}} \tilde{\delta\varphi} \Delta \tilde{\delta\varphi} + \omega_{\tilde{\psi}} \tilde{\psi} \Delta \tilde{\psi} \\ & + d_{\tilde{\psi}} (\Delta \tilde{\psi})^2 + d_{\tilde{\varphi}} (\Delta \tilde{\delta\varphi})^2 + \tilde{d}_{\tilde{\psi}} \Delta \tilde{\psi} \Delta^2 \tilde{\psi} + \tilde{d}_{\tilde{\varphi}} \Delta \tilde{\delta\varphi} \Delta^2 \tilde{\delta\varphi}. \end{aligned} \quad (4.13)$$

The coefficients can be found in Appendix B. Thus, the degrees of freedom have a positive sign for the kinetic terms in the action and there are no ghosts. In the range $1/3 < \lambda < 1$ we must use the rescalings

$$\tilde{\psi} \equiv \sqrt{\frac{-4(3\lambda-1)}{\lambda-1}}\psi, \quad \tilde{\delta\varphi} \equiv \frac{\delta\varphi}{\chi} \quad (4.14)$$

which changes the sign of the kinetic term of ψ . Hence, in this range of values of λ the extra gravitational degree of freedom is ghost-like.

C. Number of physical degrees of freedom

In [5] perturbations in the non-projectable version of Hořava-Lifshitz gravity were analyzed, and it was shown that not all the degrees of freedom which naïvely appear in an expansion similar to Eq. (4.10) are really dynamical. Indeed, after introducing the Sasaki-Mukhanov [48, 49] variable ζ defined as

$$\zeta \equiv -\psi - \frac{H}{\dot{\varphi}_0}\delta\varphi \quad (4.15)$$

and substituting for $\delta\varphi$ in terms of ζ , there remained only one variable and it entered the Lagrangian with a proper kinetic term. Thus, the potentially dangerous degree of freedom was in fact not dynamical. The same “trick” is not successful in the present case. Written in terms of ζ , the Lagrangian takes the following form:

$$\begin{aligned} \mathcal{L}_2^{(s)}[\psi, \zeta] = & \frac{\dot{\varphi}_0^2}{H^2\chi^2} \frac{\dot{\zeta}^2}{2} + \left[\frac{4(3\lambda-1)}{(\lambda-1)} + \frac{\dot{\varphi}_0^2}{H^2\chi^2} \right] \frac{\dot{\psi}^2}{2} + f_\zeta \zeta \dot{\zeta} + f_\psi \psi \dot{\psi} + f_{\zeta\psi} \psi \dot{\zeta} + \tilde{f}_{\zeta\psi} \zeta \dot{\psi} + g_{\zeta\psi} \dot{\zeta} \dot{\psi} + \\ & - m_\psi^2 \psi^2 - m_\zeta^2 \zeta^2 - m_{\zeta\psi}^2 \zeta \psi \\ & + \omega_\zeta \zeta \Delta \zeta + \omega_\psi \psi \Delta \psi + \omega_{\zeta\psi} \psi \Delta \zeta + \tilde{\omega}_{\zeta\psi} \zeta \Delta \psi + \\ & + d_\psi (\Delta \psi)^2 + d_\zeta (\Delta \zeta)^2 + d_{\zeta\psi} \Delta \zeta \Delta \psi + \tilde{d}_{\zeta\psi} \Delta \zeta \Delta^2 \psi + \tilde{d}_\psi \Delta \psi \Delta^2 \psi + \tilde{d}_\zeta \Delta \zeta \Delta^2 \zeta. \end{aligned} \quad (4.16)$$

Once again, the various coefficients are listed in Appendix C. Observe that, even in absence of any matter field, ψ is still a dynamical (gravitational) degree of freedom.

We wish to emphasize the fact that - as opposed to the situation in the non-projectable version - in the projectable version of HL gravity it is not possible to reduce to one the number of physical degrees of freedom, in agreement with the results of the general analysis of [37] and with the conclusions reached in many other papers in which perturbations around Minkowski background are analyzed. Note that the difference in the number of physical degrees of freedom of linear cosmological perturbations between the non-projectable version of HL gravity (analyzed in [5]) and the projectable version analyzed here is in complete agreement with the analysis made in [37] based on the classification of constraints in the Hamiltonian formalism.

D. Mass eigenvalues and discussion on tachyonic instabilities

We now want to investigate the issue of tachyonic (classical) instabilities. We do this by looking at the signs of the eigenvalues of the mass matrix. In general it is difficult to diagonalize the mass matrix. Hence, we will specialize to a couple of cases, both of them with $\Lambda = 0$ and for a potential $V_0(\varphi_0) = m^2 \varphi_0^2/2$. The first example will be a static field, the second a field oscillating around $\varphi_0 = 0$.

1. Static field and $\Lambda = 0$

Setting $\varphi_0 = x\chi$, where x is a dimensionless constant, the mass terms in Appendix B read as follows:

$$m_\psi^2 = -\frac{5}{8} \frac{\lambda-1}{3\lambda-1} m^2 x^2 \quad (4.17a)$$

$$m_{\tilde{\varphi}}^2 = \frac{m^2}{2} \quad (4.17b)$$

$$m_{\tilde{\varphi}\psi}^2 = -\frac{3}{2} \sqrt{\frac{\lambda-1}{3\lambda-1}} m^2 x. \quad (4.17c)$$

The mass matrix defined as

$$\widetilde{M}_{\tilde{\varphi}\tilde{\psi}}^2 \equiv \begin{pmatrix} m_{\tilde{\varphi}}^2 & m_{\tilde{\varphi}\tilde{\psi}}^2/2 \\ m_{\tilde{\varphi}\tilde{\psi}}^2/2 & m_{\tilde{\psi}}^2 \end{pmatrix} \quad (4.18)$$

can be easily diagonalized and its eigenvalues are

$$\widetilde{M}_+^2 = \frac{m^2}{16(3\lambda-1)} \left[4(3\lambda-1) - 5x^2(\lambda-1) + \sqrt{16(3\lambda-1)^2 + 25x^4(\lambda-1)^2 + 184x^2(\lambda-1)(3\lambda-1)} \right] \quad (4.19a)$$

$$\widetilde{M}_-^2 = \frac{m^2}{16(3\lambda-1)} \left[4(3\lambda-1) - 5x^2(\lambda-1) - \sqrt{16(3\lambda-1)^2 + 25x^4(\lambda-1)^2 + 184x^2(\lambda-1)(3\lambda-1)} \right]. \quad (4.19b)$$

In both of the ranges $\lambda > 1$ and $\lambda < 1/3$ one eigenvalue is positive (\widetilde{M}_+^2) whereas the other one is negative for any value of the scalar field. Thus, the extra scalar metric degree of freedom has a tachyonic instability in these regions of λ (the ones which do not suffer from the ghost problem), as is also known from previous works which considered fluctuations in a theory without matter.

In terms of the variables ζ, ψ we obtain the following eigenvalues:

$$M_+^2 = -\frac{m^2}{4} \left[12(3\lambda-1) + 5x^2 + \sqrt{25x^4 + 144(3\lambda-1)^2} \right] \quad (4.20a)$$

$$M_-^2 = -\frac{m^2}{4} \left[12(3\lambda-1) + 5x^2 - \sqrt{25x^4 + 144(3\lambda-1)^2} \right] \quad (4.20b)$$

which are both negative for any $\lambda > 1/3$ and for any x .

2. Oscillating field and $\Lambda = 0$

We set $\varphi_0 = A \cos(mt)$ and then average over field oscillations as follows:

$$\langle f(t) \rangle \equiv \frac{m}{2\pi} \int_{-\pi/m}^{\pi/m} dt f(t). \quad (4.21)$$

For instance, we get the following result for the Hubble parameter,

$$\langle H^2 \rangle = \frac{1}{3\chi^2(3\lambda-1)} \frac{m^2 A^2}{2}, \quad (4.22)$$

while the mass terms become

$$\langle m_{\tilde{\psi}}^2 \rangle = -\frac{13}{6} \frac{\lambda-1}{3\lambda-1} \frac{m^2 A^2}{\chi^2} \quad (4.23a)$$

$$\langle m_{\tilde{\varphi}}^2 \rangle = \frac{m^2}{8} \left(4 - \frac{1}{\lambda-1} \frac{A^2}{\chi^2} \right) \quad (4.23b)$$

$$\langle m_{\tilde{\psi}\tilde{\varphi}}^2 \rangle = 0. \quad (4.23c)$$

We see that $\langle m_{\tilde{\psi}}^2 \rangle$ is negative in both of the regions $\lambda > 1$ and $\lambda < 1/3$ which are ghost-free. Thus, $\tilde{\psi}$ displays tachyonic instability.

In terms of ζ and ψ we get

$$\langle m_{\zeta}^2 \rangle = -\frac{9m^2 A^2 (7\lambda-3)}{32(\lambda-1)\chi^2} \quad (4.24a)$$

$$\langle m_{\psi}^2 \rangle = -\frac{m^2 A^2 (131\lambda-95)}{32(\lambda-1)\chi^2} \quad (4.24b)$$

$$\langle m_{\zeta\psi}^2 \rangle = -\frac{9m^2 A^2 (5\lambda-1)}{16(\lambda-1)\chi^2} \quad (4.24c)$$

and the following eigenvalues, which are both negative for any $\lambda > 1$:

$$M_+^2 = -\frac{m^2 A^2}{32(\lambda-1)\chi^2} \left(97\lambda - 61 + \sqrt{1237 - 3122\lambda + 3181\lambda^2} \right), \quad (4.25)$$

$$M_-^2 = -\frac{m^2 A^2}{32(\lambda-1)\chi^2} \left(97\lambda - 61 - \sqrt{1237 - 3122\lambda + 3181\lambda^2} \right). \quad (4.26)$$

V. CONCLUSIONS

In this paper we have studied linear cosmological perturbations in the projectable version of Hořava-Lifshitz gravity. We find that - unlike what happens in the non-projectable version - there are two physical degrees of freedom for scalar metric fluctuations (in addition to possible entropy modes). The extra degree of freedom is either ghost-like (for $1/3 < \lambda < 1$) or tachyonic (for $\lambda < 1/3$ and $\lambda > 1$). Hence, linear cosmological perturbation theory is sick for all values of λ except for the value $\lambda = 1$ corresponding to Einstein gravity.

Turning to the “strong coupling problem” first discussed in [14], we notice that - again in contrast to what happens in the non-projectable version of HL gravity - the strong coupling problem in the limit $\lambda \rightarrow 1$ manifests itself in the divergence of the coefficients in the second order action for the extra degree of freedom. Tracing back the origin of this divergence, we see that it comes from the factor $(\lambda-1)$ which multiplies the variable B in the super-momentum constraint. The super-momentum constraint is used to solve for B , and hence a divergence arises in the limit $\lambda \rightarrow 1$. In General Relativity, B is a pure gauge mode and can be set to zero from the outset. In the non-projectable version of HL gravity a combination of the super-momentum and Hamiltonian constraints is used to solve for B , and in this combination the limit $\lambda \rightarrow 1$ does not lead to any strong coupling instability if $H \neq 0$.

In conclusion, we find that cosmological perturbation theory in the projectable version of HL gravity suffers from the presence of an extra unwanted degree of freedom for scalar metric fluctuations. To make matters worse, this degree of freedom is either ghost-like or else has a tachyonic instability (depending on the value of λ). Furthermore, in the limit $\lambda \rightarrow 1$ a strong coupling instability arises. In work in progress we are studying linear cosmological perturbations in the “safe” version of HL gravity proposed in [19], with the goal of checking if any of the problems discussed above are cured.

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Appendix A: Coefficients in Eq. (4.10)

$$f_\psi = -6H(3\lambda-1), \quad f_{\varphi\psi} = -3\frac{\dot{\varphi}_0}{\chi^2}, \quad \tilde{f}_{\varphi\psi} = -\frac{3\lambda-1}{\lambda-1}\frac{\dot{\varphi}_0}{\chi^2} \quad (A1)$$

$$m_\psi^2 = -\frac{39}{2}(3\lambda-1)H^2 + 3\Lambda + \frac{3}{2}\frac{V_0(\varphi_0)}{\chi^2} - \frac{3}{4}\frac{\dot{\varphi}_0^2}{\chi^2} \quad (A2a)$$

$$m_\varphi^2 = -\frac{1}{4(\lambda-1)}\frac{\dot{\varphi}_0^2}{\chi^4} + \frac{1}{2}\frac{V_{0,\varphi\varphi}(\varphi_0)}{\chi^2} \quad (A2b)$$

$$m_{\psi\varphi}^2 = -3\frac{V_{0,\varphi}(\varphi_0)}{\chi^2} \quad (A2c)$$

$$w_\psi = -2, \quad w_\varphi = \frac{V_1(\varphi_0)}{\chi^2} \quad (A3)$$

$$d_\varphi = -\frac{V_{4,\varphi}(\varphi_0)}{\chi^2} - \frac{V_2(\varphi_0)}{\chi^2}, \quad d_\psi = -16\frac{g_2}{\chi^2} - 6\frac{g_3}{\chi^2}, \quad \tilde{d}_\psi = 6\frac{g_8}{\chi^4} - 16\frac{g_7}{\chi^4}, \quad \tilde{d}_\varphi = -\frac{V_6(\varphi_0)}{\chi^2} \quad (A4)$$

Appendix B: Coefficients in Eq. (4.13)

$$f_{\tilde{\psi}} = -\frac{3}{2}H(\lambda-1), \quad f_{\tilde{\varphi}\tilde{\psi}} = -\frac{3}{2}\sqrt{\frac{\lambda-1}{3\lambda-1}}\frac{\dot{\varphi}_0}{\chi}, \quad \tilde{f}_{\tilde{\varphi}\tilde{\psi}} = -\frac{1}{2}\sqrt{\frac{3\lambda-1}{\lambda-1}}\frac{\dot{\varphi}_0}{\chi} \quad (B1)$$

$$m_{\tilde{\psi}}^2 = -\frac{3}{16}\frac{\lambda-1}{3\lambda-1}\frac{\dot{\varphi}_0^2}{\chi^2} + \frac{3}{8}\frac{\lambda-1}{3\lambda-1}\frac{V_0(\varphi_0)}{\chi^2} - \frac{39}{8}(\lambda-1)H^2 + \frac{3}{4}\frac{\lambda-1}{3\lambda-1}\Lambda \quad (B2a)$$

$$m_{\tilde{\delta}\varphi}^2 = -\frac{1}{4(\lambda-1)}\frac{\dot{\varphi}_0^2}{\chi^2} + \frac{1}{2}V_{0,\varphi\varphi}(\varphi_0) \quad (B2b)$$

$$m_{\tilde{\psi}\tilde{\delta}\varphi}^2 = -\frac{3}{2}\sqrt{\frac{\lambda-1}{3\lambda-1}}\frac{V_{0,\varphi}(\varphi_0)}{\chi} \quad (B2c)$$

$$w_{\tilde{\psi}} = -\frac{1}{2}\frac{\lambda-1}{3\lambda-1}, \quad \omega_{\tilde{\varphi}} = V_1(\varphi_0) \quad (B3)$$

$$d_{\tilde{\varphi}} = -V_{4,\varphi}(\varphi_0) - V_2(\varphi_0), \quad d_{\tilde{\psi}} = -4\frac{\lambda-1}{3\lambda-1}\frac{g_2}{\chi^2} - \frac{3}{2}\frac{\lambda-1}{3\lambda-1}\frac{g_3}{\chi^2}, \quad \tilde{d}_{\tilde{\psi}} = -4\frac{\lambda-1}{3\lambda-1}\frac{g_7}{\chi^4} + \frac{3}{2}\frac{\lambda-1}{3\lambda-1}\frac{g_8}{\chi^4}, \quad \tilde{d}_{\tilde{\varphi}} = -V_6(\varphi_0) \quad (B4)$$

Appendix C: Coefficients in Eq. (4.16)

In what follows it is

$$F(\varphi_0) \equiv 2\Lambda\chi^2 - \dot{\varphi}_0^2 + V_0(\varphi_0) \quad (C1)$$

$$\begin{aligned} m_{\zeta}^2 = & -\frac{1}{4(\lambda-1)}\frac{\dot{\varphi}_0^4}{\chi^4H^2} - \frac{1}{2}\frac{\dot{\varphi}_0^2}{\chi^2} + \frac{1}{2}\frac{V_{0,\varphi\varphi}\dot{\varphi}_0^2}{\chi^2H^2} + \\ & -\frac{1}{3(3\lambda-1)}\frac{\dot{\varphi}_0}{\chi^4H^3}(3H\dot{\varphi}_0 + V_{0,\varphi})F(\varphi_0) - \frac{1}{2}\frac{(3H\dot{\varphi}_0 + V_{0,\varphi})^2}{\chi^2H^2} + \frac{\dot{\varphi}_0}{\chi^2H}(3H\dot{\varphi}_0 + V_{0,\varphi}) \\ & -\frac{1}{18(3\lambda-1)^2}\frac{\dot{\varphi}_0^2}{\chi^6H^4}F(\varphi_0)^2 + \frac{1}{3(3\lambda-1)}\frac{\dot{\varphi}_0^2}{\chi^4H^2}F(\varphi_0) \end{aligned} \quad (C2a)$$

$$\begin{aligned} m_{\psi}^2 = & -\frac{39}{2}H^2(3\lambda-1) + 3\Lambda + 3\frac{\dot{\varphi}_0}{H\chi^2}V_{0,\varphi} - \frac{1}{4(\lambda-1)}\frac{\dot{\varphi}_0^4}{\chi^4H^2} + \frac{1}{2}\frac{V_{0,\varphi\varphi}\dot{\varphi}_0^2}{\chi^2H^2} - \frac{17}{4}\frac{\dot{\varphi}_0^2}{\chi^2} + \frac{3}{2}\frac{V_0(\varphi_0)}{\chi^2} \\ & -\frac{1}{3(3\lambda-1)}\frac{\dot{\varphi}_0}{\chi^4H^3}(3H\dot{\varphi}_0 + V_{0,\varphi})F(\varphi_0) - \frac{1}{2}\frac{(3H\dot{\varphi}_0 + V_{0,\varphi})^2}{\chi^2H^2} + 4\frac{\dot{\varphi}_0}{\chi^2H}(3H\dot{\varphi}_0 + V_{0,\varphi}) \\ & -\frac{1}{18(3\lambda-1)^2}\frac{\dot{\varphi}_0^2}{\chi^6H^4}F(\varphi_0)^2 + \frac{4}{3(3\lambda-1)}\frac{\dot{\varphi}_0^2}{\chi^4H^2}F(\varphi) \end{aligned} \quad (C2b)$$

$$\begin{aligned} m_{\zeta\psi}^2 = & +\frac{5}{3(3\lambda-1)}\frac{\dot{\varphi}_0^2}{\chi^4H^2}F(\varphi_0) - \frac{2}{3(3\lambda-1)}\frac{\dot{\varphi}_0}{\chi^4H^3}(3H\dot{\varphi}_0 + V_{0,\varphi})F(\varphi_0) + \\ & -\frac{1}{2(\lambda-1)}\frac{\dot{\varphi}_0^4}{\chi^4H^2} - 4\frac{\dot{\varphi}_0^2}{\chi^2} + \frac{V_{0,\varphi\varphi}\dot{\varphi}_0^2}{\chi^2H^2} - \frac{1}{\chi^2H^2}(3H\dot{\varphi}_0 + V_{0,\varphi})^2 + \\ & +\frac{5\dot{\varphi}_0}{\chi^2H}(3H\dot{\varphi}_0 + V_{0,\varphi}) - \frac{1}{9(3\lambda-1)^2}\frac{\dot{\varphi}_0^2}{\chi^6H^4}F(\varphi_0)^2 + 3\frac{\dot{\varphi}_0V_{0,\varphi}}{H\chi^2} \end{aligned} \quad (C2c)$$

$$f_\zeta = -\frac{1}{3(3\lambda-1)} \frac{\dot{\varphi}_0^2}{\chi^4 H^3} F(\varphi_0) + \frac{\dot{\varphi}_0^2}{\chi^2 H} - \frac{\dot{\varphi}_0}{\chi^2 H^2} (3H\dot{\varphi}_0 + V_{0,\varphi}) \quad (\text{C3a})$$

$$f_\psi = -\frac{1}{3(3\lambda-1)} \frac{\dot{\varphi}_0^2}{\chi^4 H^3} F(\varphi_0) + \left(4 + \frac{3\lambda-1}{\lambda-1}\right) \frac{\dot{\varphi}_0^2}{\chi^2 H} - \frac{\dot{\varphi}_0}{\chi^2 H^2} (3H\dot{\varphi}_0 + V_{0,\varphi}) - 6H(3\lambda-1) \quad (\text{C3b})$$

$$f_{\zeta\psi} = f_\zeta \quad (\text{C3c})$$

$$\tilde{f}_{\zeta\psi} = -\frac{1}{3(3\lambda-1)} \frac{\dot{\varphi}_0^2}{\chi^4 H^3} F(\varphi_0) + \left(1 + \frac{3\lambda-1}{\lambda-1}\right) \frac{\dot{\varphi}_0^2}{\chi^2 H} - \frac{\dot{\varphi}_0}{\chi^2 H^2} (3H\dot{\varphi}_0 + V_{0,\varphi}) \quad (\text{C3d})$$

$$g_{\zeta\psi} = \frac{\dot{\varphi}_0^2}{\chi^2 H^2} \quad (\text{C3e})$$

$$\omega_\zeta = \frac{V_1(\varphi_0)\dot{\varphi}_0^2}{\chi^2 H^2} \quad (\text{C4a})$$

$$\omega_\psi = \omega_\zeta - 2 \quad (\text{C4b})$$

$$\tilde{\omega}_{\zeta\psi} = \omega_\zeta \quad (\text{C4c})$$

$$\omega_{\zeta\psi} = \omega_\zeta \quad (\text{C4d})$$

$$d_\psi = -4 \frac{\dot{\varphi}_0 V_4(\varphi_0)}{\chi^2 H} + 4 \frac{V_4(\varphi_0)\dot{\varphi}_0}{\chi^2 H} - \frac{[V_{4,\varphi}(\varphi_0) + V_2(\varphi_0)]\dot{\varphi}_0^2}{\chi^2 H^2} - \frac{2}{\chi^2} (8g_2 + 3g_3) \quad (\text{C5a})$$

$$d_{\zeta\psi} = -4 \frac{\dot{\varphi}_0 V_4(\varphi_0)}{\chi^2 H} + 4 \frac{V_4(\varphi_0)\dot{\varphi}_0}{\chi^2 H} - 2 \frac{[V_{4,\varphi}(\varphi_0) + V_2(\varphi_0)]\dot{\varphi}_0^2}{\chi^2 H^2} \quad (\text{C5b})$$

$$d_\zeta = -\frac{[V_{4,\varphi}(\varphi_0) + V_2(\varphi_0)]\dot{\varphi}_0^2}{\chi^2 H^2} \quad (\text{C5c})$$

$$\tilde{d}_{\zeta\psi} = -2 \frac{V_6(\varphi_0)\dot{\varphi}_0^2}{\chi^2 H^2} \quad (\text{C5d})$$

$$\tilde{d}_\psi = -\frac{V_6(\varphi_0)\dot{\varphi}_0^2}{\chi^2 H^2} - \frac{2}{\chi^4} (8g_7 - 3g_8) \quad (\text{C5e})$$

$$\tilde{d}_\zeta = -\frac{V_6(\varphi_0)\dot{\varphi}_0^2}{\chi^2 H^2} \quad (\text{C5f})$$

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